American University of Beirut

STAT 230

Introduction to Probability and Random Variables

Summer 2009

quiz # 2 - solution part 2

1. If X is uniformly distributed over $(-1, 1)$. Find $P(|X| > \frac{1}{2})$.

$P(|X| > \frac{1}{2}) = \frac{1}{2}$

2. Let X be a random variable with pdf $f(x) = 3x^2$, $0 < x < 1$. Find $E(X^n)$, then deduce $Var(X)$.

$E(X^n) = \displaystyle\int_0^1 3x^{n+2}dx = \frac{3}{n+3}$, then $Var(X) = E(X^2) - (E(X))^2 = \frac{3}{5} - \frac{9}{16} = \frac{3}{80}$.

3. Let X be a random variable with pdf

$$f(x) = \begin{cases} x+1 & -1 < x < 0 \\ 1-x & 0 < x < 1 \end{cases}$$

Find $F_X(x)$, the cdf of X.

$$F_X(x) = \begin{cases} 0 & x < -1 \\ \frac{x^2}{2} + x + \frac{1}{2} & -1 < x < 0 \\ \frac{1}{2} + x - \frac{x^2}{2} & 0 < x < 1 \\ 1 & x > 1 \end{cases}$$

4. The time (in hours) required to repair a machine is an exponentially distributed random variable with parameter $\lambda = 1/2$. Find the probability that a repair time takes at least 10 hours given that its duration exceeds 9 hours.

$X \rightsquigarrow f(x) = \frac{1}{2}\, e^{-\frac{x}{2}}$, $0 < x < \infty$.

$P(X \geq 10 | X \geq 9) = P(X \geq 1)$ (memoryless property of the exponential distribution).

$P(X \geq 1) = \displaystyle\int_0^\infty \frac{1}{2}\, e^{-\frac{x}{2}}\, dx = e^{-\frac{1}{2}}$

5. Let X be a random variable with pdf $f(x) = Cx^3e^{-2x}$, $0 < x < \infty$. Find the value of C.

$C = \frac{1}{\Gamma(4) \times (1/2)^4}$ by comparison with a Gamma distribution, then $C = 8/3$

6. Let X be a random variable with pdf $f(x) = 3x^2$, $0 < x < 1$. Find the pdf of $Y = -6 \ln X$.

$y = -6 \ln x$ is one-to-one; the interval $]0, 1[$ is mapped into $]0, \infty[$.

$x = e^{-y/6} = g^{-1}(y)$, and $(g^{-1}(y))' = -\frac{1}{6}\, e^{-y/6}$, and then

$h(y) = | -\frac{1}{6}\, e^{-y/6}| \times 3(e^{-y/6})^2 = \frac{1}{2}\, e^{-y/2}$, $0 < y < \infty$